Show all work neatly and systematically for full credit. Total points: 100
When Construct Confidence Interval, please make sure you have the critical value, margin of error, then construct the confidence Interval.
When perform a hypothesis testing, please make sure to show all the steps as learned in class.

<2, 2, 2> Find the indicated critical value(s).
1) Find the critical value \( z_{\alpha/2} \) that corresponds to a 91\% confidence level.
\[ z_{0.095} = z_{0.045} = z_{0.045} = 1.695 \] 

2) Find the critical t-value that corresponds to 95\% confidence and \( n = 16 \).
\[ t_{\text{critical}} = 1.753 \]

3) Find the critical values, \( \chi^2_{1 - \alpha/2} \) and \( \chi^2_{\alpha/2} \), for 90\% confidence and \( n = 15 \).
\[ \chi^2_{1 - 0.05} = \chi^2_{0.95} = 6.634 \]
\[ \chi^2_{0.05} = 23.685 \]

<6> Express the null hypothesis and the alternative hypothesis in symbolic form. Use the correct symbol (\( \mu \), \( p \), \( \sigma \)) for the indicated parameter.
4) (a) An entomologist writes an article in a scientific journal which claims that fewer than 16 in ten thousand male fireflies are unable to produce light due to a genetic mutation. Use the parameter \( p \), the true proportion of fireflies unable to produce light.
\[ P = \frac{16}{10000} = 0.0016 \]
\[ H_0 : P = 0.0016 \]
\[ H_1 : P < 0.0016 \]

(b) The owner of a football team claims that the average attendance at games is over 68,800, and he is therefore justified in moving the team to a city with a larger stadium.
\[ M > 68800 \]
\[ H_0 : M = 68800 \]
\[ H_1 : M > 68800 \]

<6> Use the given information to find the P-value.
5) (a). The test statistic in a right-tailed test is \( z = 1.43 \).
\[ P = 1 - 0.9236 = 0.0764 \]

(b). The test statistic in a left-tailed test is \( z = -1.83 \).
\[ P = 0.0336 \]
6) 8 points A survey of 865 voters in one state reveals that 408 favor approval of an issue before the legislature. Construct the 95% confidence interval for the true proportion of all voters in the state who favor approval.

Point estimate: \( \hat{p} = \frac{408}{865} \)

Critical Value: \( Z_{0.025} = 1.96 \) (critical value)

Margin of Error: \( E = Z_{0.025} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96 \sqrt{\frac{0.47168 \cdot 0.52832}{865}} \)

Confidence Interval: \( 0.43841 < p < 0.50495 \)

7) 5 points A doctor at a local hospital is interested in estimating the birth weight of infants. How large a sample must she select if she desires to be 98% confident that the true mean is within 2 ounces of the sample mean? The standard deviation of the birth weights is known to be 8 ounces.

\[ n = \left[ \frac{Z_{0.01} \cdot \sigma}{E} \right]^2 = \left[ \frac{2.326 \cdot 8}{2} \right]^2 = 86.49 \]

\( E = 2 \)

\( n \) should be at least 87 sample size

8) 10 points A recent survey showed that in a sample of 100 elementary school teachers, 15 were single. In a sample of 180 high school teachers, 36 were single. Is the proportion of high school teachers who were single greater than the proportion of elementary teachers who were single? Use \( \alpha = 0.01 \).

\( \hat{p}_1 = \frac{15}{100} = 0.15 \)

\( \hat{p}_2 = \frac{36}{180} = 0.2 \)

\( \hat{p}_1 - \hat{p}_2 = -0.05 \)

\( E = Z_{0.005} \sqrt{\frac{\hat{p}_1 \cdot \hat{q}_1}{n_1} + \frac{\hat{p}_2 \cdot \hat{q}_2}{n_2}} = 2.575 \sqrt{\frac{0.15 \cdot 0.85 + 0.2 \cdot 0.8}{100 + 180}} \)

\( E = 0.11978 \)

\( \hat{q}_1 = 1 - 0.15 = 0.85 \)

\( \hat{q}_2 = 1 - 0.2 = 0.8 \)

\( -0.05 - 0.11978 < \hat{p}_1 - \hat{p}_2 < -0.05 + 0.11978 \)

\( \Rightarrow -0.16978 < \hat{p}_1 - \hat{p}_2 < 0.06978 \)

The interval contains 0, meaning that there is no significant difference between two proportions, so the proportion of high school teachers who were single is not greater than the proportion of elementary teachers who were single.
9) <8 points> A random sample of 10 parking meters in a resort community showed the following incomes for a day. Assume the incomes are normally distributed. Find the 95% confidence interval for the true mean.

\[
\begin{align*}
\$3.60 & \quad \$4.50 & \quad \$2.80 & \quad \$6.30 & \quad \$2.60 & \quad \$5.20 & \quad \$6.75 & \quad \$4.25 & \quad \$8.00 & \quad \$3.00
\end{align*}
\]

Point Estimate: \( \bar{x} = 4.7 \)

Critical Values: \( z_{0.025} = 2.262 \)

Margin of Error: \( E = 1.3172 \)

Confidence Interval: \( 3.3882 < \mu < 6.0172 \)

10) <5 points> A researcher at a major clinic wishes to estimate the proportion of the adult population of the United States that has sleep deprivation. How large a sample is needed in order to be 99% confident that the sample proportion will not differ from the true proportion by more than 4%?

\[
E = 0.04 \quad \Rightarrow \quad n = \frac{Z_{0.025}^2 \cdot 0.25}{0.04^2} = 1036.035 \approx 1037
\]

n should be at least 1037 sample size

11) <8 points> A student randomly selects 10 paperbacks at a store. The mean price is $8.75 with a standard deviation of $1.50. Construct a 95% confidence interval for the population standard deviation. Assume the data are normally distributed.

Critical Value: \( \chi^2_{0.025} = 19.023 \)

Confidence Interval: \( 1.5 < \sigma < 2.73861 \)

\[
\left( \frac{10-1}{19.023} \right)^{1.5^2} < \sigma < \left[ \frac{(10-1) \cdot 1.5^2}{2.7} \right]^{1/2}
\]

\[
\Rightarrow 1.03175 < \sigma < 2.73861
\]
12) <10 points> A supplier of digital memory cards claims that no more than 1% of the cards are defective. In a random sample of 600 memory cards, it is found that 3% are defective, but the supplier claims that this is only a sample fluctuation. At the 0.01 level of significance, test the supplier's claim that no more than 1% are defective.

\[ \hat{p} = 0.03, \hat{q} = 0.97, n = 600, np = 18 > 5, nq = 582 > 5 \quad p \leq 0.01 \]

\[ H_0: p = 0.01 \quad \checkmark \]

\[ H_1: p > 0.01 \]

\[ \alpha = 0.01, Z = 2.325 \text{ (critical value)} \]

\[ \begin{align*}
\varepsilon &= \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}\hat{q}}{n}}} \\
&= \frac{0.03 - 0.01}{\sqrt{\frac{0.01 \times 0.99}{600}}} \approx 4.924 \\
4.924 &> 2.325 \\
\text{Reject } H_0.
\end{align*} \]

There is sufficient evidence to warrant rejection of the claim that no more than 1% are defective.

13) <8 points> In a recent survey of drinking laws, a random sample of 1000 women showed that 65% were in favor of increasing the legal drinking age. In a random sample of 1000 men, 60% favored increasing the legal drinking age. Construct a 95% confidence interval for \( p_1 - p_2 \).

\[ \text{Critical Value: } 1.96 \]  
\[ \text{Margin of Error: } 0.04238 \]  
\[ E = \frac{Z_{0.025}}{\sqrt{n_1} + \frac{n_2}{n_2}} = 1.96 \left( \frac{0.65 \times 0.35}{1000} \right) = 0.04238 \]

\[ \hat{p}_1 = 0.65, \hat{q}_1 = 1 - 0.65 = 0.35, n_1 = 1000 \]
\[ \hat{p}_2 = 0.6, \hat{q}_2 = 1 - 0.6 = 0.4, n_2 = 1000 \]

\[ \text{Confidence Interval: } (\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E \]

\[ 0.00762 < p_1 - p_2 < 0.09238 \]
14) <10 points> A manufacturer claims that the mean lifetime of its lithium batteries is 1000 hours. A homeowner selects 25 of these batteries and finds the mean lifetime to be 990 hours with a standard deviation of 80 hours. Test the manufacturer's claim. Use $\alpha = 0.05$.

$$
\begin{align*}
H_0 &: \mu = 1000 \\
H_1 &: \mu \neq 1000 \\
\alpha &= 0.05, \quad t = \pm 2.064 \\
\frac{t}{s / \sqrt{n}} &= \frac{990 - 1000}{80 / \sqrt{25}} = -0.625
\end{align*}
$$

$-2.064 < -0.625 < 2.064$

Fail to reject $H_0$.

There is not sufficient evidence to warrant rejection of the claim that the mean lifetime of its lithium batteries is 1000 hours.

15) <10 points> A statistics professor at an all-men's college determined that the standard deviation of men's heights is 2.5 inches. The professor then randomly selected 41 female students from a nearby all-female college and found the standard deviation to be 2.9 inches. Test the professor's claim that the standard deviation of female heights is greater than 2.5 inches. Use $\alpha = 0.01$.

$$
\begin{align*}
H_0 &: \sigma = 2.5 \\
H_1 &: \sigma > 2.5 \\
\alpha &= 0.01, \quad \chi^2 = 63.691 \\
\chi^2 &= \frac{(n-1)s^2}{\sigma^2} = \frac{(41-1)2.9^2}{2.5^2} = 57.824
\end{align*}
$$

Fail to reject $H_0$.

There is not sufficient sample evidence to support the claim that the standard deviation of female heights is greater than 2.5 inches.