Fall 2018 Math227 Test #3 Name KESHAV MISHRA 99

Show all work neatly and systematically for full credit. Total points:101.
Note: for hypothesis testing and confidence interval, make sure to show all steps.

1) (10) Leakage from underground fuel tanks has been a source of water pollution. In a random sample of 107 gasoline stations, 18 were found to have at least one leaking underground tank. Construct a 95% confidence interval for the proportion of gasoline stations with at least one leaking underground tank.

Which distribution use? Explain. normal distribution ($n\hat{p} \geq 5$ & $n\hat{q} \geq 5$)

$\hat{p} = 0.168$; $\hat{q} = 0.832$ $n\hat{p} = 17.976$; $n\hat{q} = 89.024$

Critical value: $Z_{\alpha/2} = 1.96$

$\alpha = 5\%$

$\alpha/2 = 0.025$

-2 Margin of Error: $E = Z_{\alpha/2} \times \sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = 1.96 \times 0.0361 = 0.0686 \approx \cancel{0.069}$

Confidence Interval: $\hat{p} - E < p < \hat{p} + E$

$= 0.099 < p < 0.237$

Conclusion sentence:

We are 95% confident that the interval from 0.099 to 0.237 contains the true proportion of gasoline stationss with atleast one leaking underground tank.

2) (5) The principal at Riverside High School would like to estimate the mean length of time each day that it takes all the buses to arrive and unload the students. How large a sample is needed if the principal would like to assert with 98% confidence that the sample mean is off by, at most, 5 minutes? Assume that s = 10 minutes based on previous studies.

Sample size $= n = \left(\frac{Z_{\alpha/2} \cdot S}{E}\right)^2$

$S = 10$ mins.

$E = 5$ mins.

$\alpha = 0.02$; $\alpha/2 = 0.01$

$Z_{\alpha/2} = 2.326$

$\therefore n = \left(\frac{2.326 \times 10}{5}\right)^2$

$= 21.64 \approx 22$

$\therefore$ He will need a sample size of 22 to assert with 98% confidence.

$E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$

$\Rightarrow \sqrt{n} = \frac{t_{\alpha/2} \cdot S}{E}$

$n = \left(\frac{Z_{\alpha/2} \cdot S}{E}\right)^2$

-2

3) (8) A research wantsto estimate the proportion of households that have broadbank Internet access. What size sample should be obtained if she wishes the estimate to be within 0.03 with 99% confidence if

a. she uses an estimate of 0.635 obtained from the National Telecommunications and Information Administration?

$\hat{p} = 0.635$; $\hat{q} = 0.365$; $\alpha/2 = 0.005$; $E = 0.03$

$$n = \frac{z_{\alpha/2}^2 \cdot \hat{p} \cdot \hat{q}}{E^2} = \frac{(2.576)^2 \times 0.635 \times 0.365}{(0.03)^2}$$

$= 1708.67 \cong 1709$ samples

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$

$\Rightarrow E^2 = z_{\alpha/2}^2 \cdot \frac{\hat{p}\hat{q}}{n}$

$\Rightarrow n = \frac{z_{\alpha/2}^2 \, \hat{p} \cdot \hat{q}}{E^2}$

b. she does not use any prior estimate?

$\hat{p} = 0.5$; $\hat{q} = 0.5$; $E = 0.03$

$$n = \frac{z_{\alpha/2}^2 \cdot \hat{p} \cdot \hat{q}}{E^2} = 1843.27 \cong 1844 \text{ samples}$$

4) (10) In a sample of 87 young adult, the average time per day spent in bed asleep was 7.06 hours and the standard deviation was 1.11 hours. Construct a 99% confidence interval for the mean time spent in bed asleep.

Which distribution use? Explain. t-distribution [$n > 30$ & σ not known]

$n = 87$; $\bar{x} = 7.06$ hrs ; $s = 1.11$ hrs.

Critical value: $t_{\alpha/2} = 2.634$

$\alpha = 0.01$, $df = 86$

Margin of Error: $E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 2.634 \times 0.119 = 0.313$

Confidence Interval: $\bar{x} - E < \mu < \bar{x} + E$

$= 6.747 < \mu < 7.373$ ✓

Conclusion sentence:
We are 99% confident that the interval from 6.747 to 7.373 contains the true population mean.

(9) Find the critical value(s).

5) a. Determine the critical value. Assume t distribution, it is a left-tailed test of a population mean at the $\alpha = 0.05$ level of significance and $n = 15$.

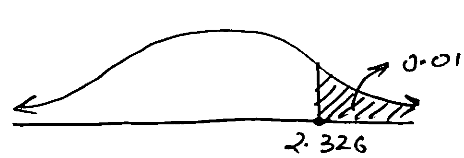
$\alpha = 0.05$; $df = 14$

$t_{crit.} = -1.761$ ✓

b. Determine the critical value. Assume normal distribution. Test the claim about the population proportion $p > 0.28$, and $\alpha = 0.01$.

Right-tailed test ; $\alpha = 0.01$

$z_{crit.} = 2.326$ ✓

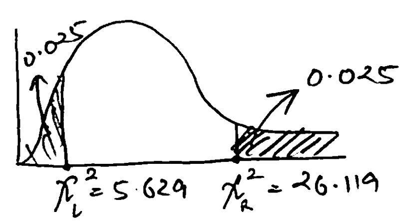


c. Determine the critical values for a two-tailed test of a population standard deviation for a sample of size $n = 15$ at the $\alpha = 0.05$ level of significance. Assume Chi-square distribution.

$df = 14$;

$\chi_R^2 = 26.119$ ✓

$\chi_L^2 = 5.629$ ✓



6) (10) A supplier of digital memory cards claims that less than 1% of the cards are defective. In a random sample of 600 memory cards, it is found that 2% are defective, but the supplier claims that this is only a sample fluctuation. At the 0.01 level of significance, test the supplier's claim that less than 1% are defective.

Claim: $p < 0.01$

$H_0: p = 0.01$
$H_1: p < 0.01$

$n = 600$
$\hat{p} = 0.02$

$np = 600 \times 0.01 = 6 \geq 5$
$nq = 600 \times 0.99 = 594 \geq 5$
} normal dist.

Test statistic,

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = 2.462$$

p-value = $P(x < 2.462)$
= 0.99
p-value > α
⇒ fail to reject H_0

$\alpha = 0.01$
Left-tailed test
$z_{crit.} = -2.32$

∴ Test statistic is not in critical region
∴ Fail to reject H_0
⇒ there is not sufficient evidence to support the claim that less than 1% digital memory cards are defective.

7) (10) A public bus company official claims that the mean waiting time for bus number 14 during peak hours is less than 10 minutes. Karen took bus number 14 during peak hours on 18 different occasions. Her mean waiting time was 7.9 minutes with a standard deviation of 1.5 minutes. At the 0.01 significance level, test the claim that the mean waiting time is less than 10 minutes.

claim: $\mu < 10$ mins

$H_0: \mu = 10$ mins.
$H_1: \mu < 10$ mins

[population ~~is~~ normal] → use t-dist.

$n = 18$
$\bar{x} = 7.9$ mins.
$s = 1.5$ mins.

Test statistic,

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = -5.939 \simeq -5.94$$

p-value = $P(t < -5.94)$
$= 8.07 \times 10^{-6}$
p-value < α ⇒

$\alpha = 0.01$; left tailed test
$df = 17$

$t_{crit} = -2.567$

Test statistic is in critical region
∴ Reject H_0
⇒ There is sufficient evidence to support the claim that the mean waiting time ~~is~~ less than 10 mins.

8) (10) A sociologist develops a test to measure attitudes about public transportation, and 27 randomly selected subjects are given the test. Their mean score is 76.2 and their standard deviation is 21.4. Construct the 95% confidence interval for the standard deviation of the scores of all subjects. Assume the population is normal.

$n = 27$ Population is normal → χ^2-dist.

$\bar{x} = 76.2$

$S = 21.4$

$C.L. = 95\%$ ✓

$\alpha = 0.05$; $df = 26$

$\chi_R^2 = \chi_{0.025}^2 = 41.923$ ✓

$\chi_L^2 = \chi_{0.975}^2 = 13.844$ ✓

C.I.,

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$= 16.853 < \sigma < 29.327$$ ✓

∴ We are 95% confident that the interval from 16.853 to 29.327 contains the true population standard deviation.

9) (10) For randomly selected adults, IQ scores are normally distributed with a standard deviation of 15. The scores of 14 randomly selected college students are listed below. Use a 0.10 significance level to test the claim that the standard deviation of IQ scores of college students is less than 15. Round the sample standard deviation to three decimal places.

115	128	107	109	116	124	135
127	115	104	118	126	129	133

Claim: $\sigma < 15$ ✓

$H_0 : \sigma = 15$

$H_1 : \sigma < 15$ ✓

$n = 14$ [normal population dist] → χ^2-dist.

$\bar{x} = 120.429 \simeq 120.43$ ✓

$s = 9.819$

Test statistic,

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = 5.571$$

P-value $= P(\chi^2 < 5.571)$
$= 0.039$ $(\alpha = 0.10)$
∵ P-value $< \alpha$
Reject H_0 ✓

$\alpha = 0.10$

$df = 13$

Left-tailed test

$\chi_{crit}^2 = 7.042$ ✓

Test statistic is in critical region

∴ Reject H_0

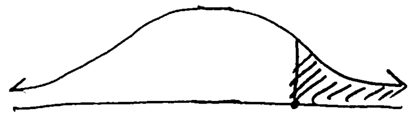
⇒ There is sufficient evidence to support the claim that the S.D. of IQ scores of college students is less than 15.

(9) Use the given information to find the P-value.

10) a. Assume normal distribution. The test statistic in a right-tailed test is z =1.52. Find p-value.

p-value $= P(z > 1.52)$

$= 0.0643$ ✓



b. Assume normal-distribution with n = 45. It is a two-tailed test. with test statistic z = -1.635. Find p-value

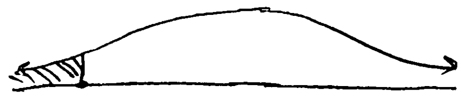
P-value $= 2 \cdot P(Z < -1.635)$

$= 0.102$ ✓

c. Assume normal distribution with n = 14. It is a left-tailed test and the test statistic is z = - 2.27. Find p-value.

p-value $= P(z < -2.27)$

$= 0.0116$ ✓



11) (10) Among 703 randomly selected workers, 420 got their jobs through networking. Use the sample data with a 0.05 significance level to test the claim that more than 50% workers get their jobs through networking.

Claim: $p > 0.5$

$H_0 : p = 0.5$ ✓

$H_1 : p > 0.5$

$n = 703$

$\hat{p} = \frac{420}{703} = 0.597$ ✓

$\hat{q} = 0.403$

$np = 703 \times 0.5 = 351.5 \geq 5$

$nq = 703 \times 0.5 = 351.5 \geq 5$ } normal dist

Test statistic,

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = 5.144$$ ✓

p-value $= P(Z > 5.144)$

$= 1.3 \times 10^{-7}$

$\alpha = 0.05$

p-value $< \alpha$

∴ Reject H_0

$\alpha = 0.05$

Right-tailed test

$z_{crit} = 1.645$ ✓

Test statistic is in critical region

∴ Reject H_0

⇒ There is sufficient evidence to support the claim that more than 50% workers get their jobs through networking. ✓