Fall 2018 Math54 Test #3 Name Waiying Lee

101

Show all work neatly and systematically for full credit. Total points:101.
Note: for hypothesis testing and confidence interval, make sure to show all steps. Make sure to write conclusion sentences.

1) (7) Leakage from underground fuel tanks has been a source of water pollution. In a random sample of $\boxed{107}$ gasoline stations, $\boxed{18}$ were found to have at least one leaking underground tank. Construct a 95% confidence interval for the proportion of gasoline stations with at least one leaking underground tank.

$n = 107$

$\hat{p} = \frac{18}{107} = 0.168$

$\hat{q} = 1 - 0.168 = 0.832$ ✓

$Z_{\frac{\alpha}{2}} = 1.96$

$E = 1.96\sqrt{\frac{(0.168)(0.832)}{107}}$ ✓

$= 0.07084$

$0.168 - 0.07084 < p < 0.168 + 0.07084$

$0.09716 < p < 0.23884$ ✓

We are 95% confident that the interval from 0.09716 to 0.23884 contain the true proportion of gasoline stations with at least one leaking underground tank. ✓

Provide an appropriate response.

2) (7) At a local store, 65 female employees were randomly selected and it was found that their mean monthly income was $625 with a standard deviation of $121.50. Seventy-five male employees were also randomly selected and their mean monthly income was found to be $667 with a standard deviation of $168.70. Test the hypothesis that male employees have a higher monthly income than female employees. Use $\alpha = 0.01$.

$n > 30$
use normal

Female	male
$n_1 = 65$	$n_2 = 75$
$\bar{X}_1 = 625$	$\bar{X}_2 = 667$
$S_1 = 121.50$	$S_2 = 168.70$
	$df = 64$

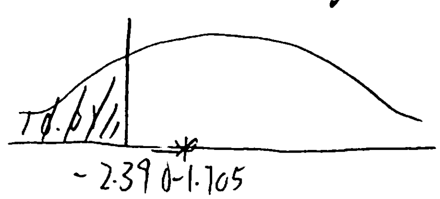
claim: $\mu_1 < \mu_2$

$H_0 = \mu_1 = \mu_2$

$H_1 = \mu_1 < \mu_2$ ✓

$t_{\frac{\alpha}{2}} = -2.390$ ✓

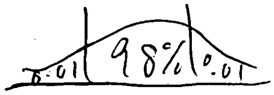
$t = \frac{625 - 667}{\sqrt{\frac{121.5^2}{65} + \frac{168.7^2}{75}}} = -1.705$ ✓



$p(t < -1.705) = tcdf(-E99, -1.705, 64)$

$= 0.0465 > 0.01$ ✓

Fail to reject H_0, there is no sufficient evidence to support the claim at $\alpha = 0.01$ that male employees have a higher monthy income than female employees.



3) (5) The principal at Riverside High School would like to estimate the mean length of time each day that it takes all the buses to arrive and unload the students. How large a sample is needed if the principal would like to assert with 98% confidence that the sample mean is off by, at most, 5 minutes? Assume that s = 10 minutes based on previous studies.

$z_{\frac{\alpha}{2}} = 2.33$

$E = 5$

$S = 10$

$n = \left[\frac{2.33(10)}{5}\right]^2$

$= 22$

The principle need 22 students

4) (3, 3) A research wantsto estimate the proportion of households that have broadbank Internet access. What size sample should be obtained if she wishes the estimate to be within 0.03 with 99% confidence if

a. she uses an estimate of 0.635 obtained from the National Telecommunications and Information Administration?

$E = 0.03$

$z_{\frac{\alpha}{2}} = 2.575$

$\hat{p} = 0.635$

$\hat{q} = 0.365$

$n = \frac{(2.575)^2(0.635)(0.365)}{0.03^2}$

$= 1707.5 \approx 1708$

b. she does not use any prior estimate?

$E = 0.03$

$z_{\frac{\alpha}{2}} = 2.575$

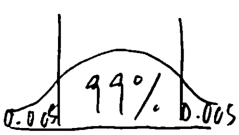
$\hat{p} = 0.5$

$\hat{q} = 0.5$

$n = \frac{(2.575)^2(0.25)}{0.03^2}$

$= 1841.8 \approx 1842$

5) (7) In a sample of 87 young adult, the average time per day spent in bed asleep was 7.06 hours and the standard deviation was 1.11 hours. Construct a 99% confidence interval for the mean time spent in bed asleep.



$n = 87, df = 86$

$\bar{x} = 7.06$

$S = 1.11$

$t_{\frac{\alpha}{2}} = 2.632$

$E = 2.632 \times \frac{1.11}{\sqrt{87}}$

$= 0.3132$

$\bar{x} - E < \mu < \bar{x} + E$

$7.06 - 0.3132 < \mu < 7.06 + 0.3132$

$6.7468 < \mu < 7.3732$

We are 99% confident that from the interval 6.7468 hours to 7.3732 hours contain the mean time that young adults spent in bed asleep.

(6) Find the critical value(s).

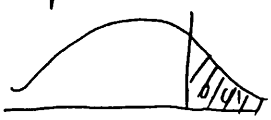
6) a. Determine the critical value. Assume t distribution, it is a left–tailed test of a population mean at the $\alpha = 0.05$ level of significance and n = 15.

$n = 15,\ df = 14$ $\qquad t_{\frac{\alpha}{2}} = -1.761$

b. Determine the critical value. Assume normal distribution. Test the claim about the population proportion $p > 0.28$, and $\alpha = 0.01$.

$\text{invNorm}(0.01, 0, 1) = 2.33$

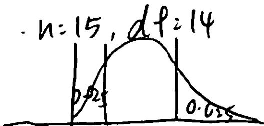
$p > 0.28$



$z_{\frac{\alpha}{2}} = 2.33$

c. Determine the critical values for a two–tailed test of a population standard deviation for a sample of size n = 15 at the $\alpha = 0.05$ level of significance. Assume Chi–square distribution.

$n = 15,\ df = 14$ $\qquad \chi^2_R = 26.119$



$\chi^2_L = 5.629$

7) (7) A supplier of digital memory cards claims that less than 1% of the cards are defective. In a random sample of 600 memory cards, it is found that 2% are defective, but the supplier claims that this is only a sample fluctuation. At the 0.01 level of significance, test the supplier's claim that less than 1% are defective.

claim: $p < 0.01$

$H_0: p = 0.01 \qquad q = 0.99$

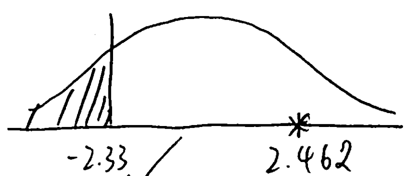
$H_1: p < 0.01$

$n = 600$

$\hat{p} = 0.02$

$\hat{q} = 0.98$

$z_{\frac{\alpha}{2}} = -2.33$



$$z = \frac{0.02 - 0.01}{\sqrt{\frac{(0.01)(0.99)}{600}}}$$

$= 2.462$

$p(z < 2.462) = \text{normalcdf}(-E99, 2.462, 0, 1)$
$= 0.9931 > 0.01$

Fail to reject H_0, there is not sufficient evidence to support the claim at $\alpha = 0.01$ that less than 1% of the cards are defective.

8) (7) A public bus company official claims that the mean waiting time for bus number 14 during peak hours is less than 10 minutes. Karen took bus number 14 during peak hours on 18 different occasions. Her mean waiting time was 7.9 minutes with a standard deviation of 1.5 minutes. At the 0.01 significance level, test the claim that the mean waiting time is less than 10 minutes.

Claim: $\mu < 10$

$H_0: \mu = 10$

$H_1: \mu < 10$

$n = 18,\ df = 17$

$\bar{x} = 7.9$

$s = 1.5$

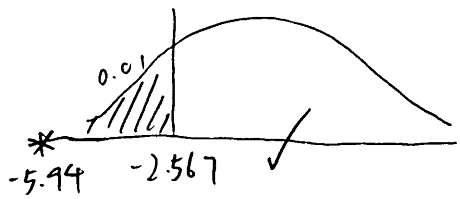
$$t = \frac{7.9 - 10}{\frac{1.5}{\sqrt{18}}} = -5.94$$

$p(t < -5.94) = tcdf(-E99, -5.94, 17)$

$= 0.000008072 \le 0.01$

Reject H_0, there is sufficient evidence to support the claim at $\alpha = 0.01$ that the mean waiting time is less than 10 minutes.

$t_{\frac{\alpha}{2}} = -2.567$



9) (7) A sociologist develops a test to measure attitudes about public transportation, and 27 randomly selected subjects are given the test. Their mean score is 76.2 and their standard deviation is 21.4. Construct the 95% confidence interval for the standard deviation, σ, of the scores of all subjects. Assume the population is normal.

$C.I = 95\%$

$n = 27$

$df = 26$

$\bar{x} = 76.2$

$s = 21.4$

$\chi^2_R = \chi^2_{0.025} = 41.923$

$\chi^2_L = \chi^2_{0.975} = 13.844$

$$\sqrt{\frac{(27-1)(21.4)^2}{41.923}} < \sigma < \sqrt{\frac{(27-1)(21.4)^2}{13.844}}$$

$$16.85 < \sigma < 29.33$$

We are 95% confident that from the interval 16.85 to 29.33 contain the standard deviation of the scores of all subjects.

10) (3, 5 points)

The reading speed of second grade students is approximately normal with a mean of 90 words per minute and a standard deviation of 10 words per minutes.

a. What is the probability $\boxed{a}$ randomly selected student will read more than 95 words per minute?

$\bar{x} = 90$

$s = 10$

$p(x > 95)$:

$z = \dfrac{95-90}{10}$

$= 0.5$

$p(z > 0.5) = \text{normalcdf}(0.5, E99, 0, 1)$

$= 0.3085$

The probability is 0.3085

b. What is the probability that a random sample of $\boxed{24}$ second grade students results in a mean reading rate of more than 95 words per minute?

$\mu_{\bar{x}} = 90$

$\sigma_{\bar{x}} = \dfrac{10}{\sqrt{24}}$

$= 2.041$

$p(x > 95)$:

$z = \dfrac{95-90}{2.041}$

$= 2.45$

$p(z > 2.45) = \text{normalcdf}(2.45, E99, 0, 1)$

$= 0.007143$

The probability is 0.007143

11) (7) For randomly selected adults, IQ scores are normally distributed with a standard deviation of 15. The scores of 14 randomly selected college students are listed below. Use a 0.10 significance level to test the claim that the standard deviation of IQ scores of college students is less than 15. Round the sample standard deviation to three decimal places.

115	128	107	109	116	124	135
127	115	104	118	126	129	133

claim: $\sigma < 15$

H_0: $\sigma = 15$

H_1: $\sigma < 15$

$n = 14$

$df = 13$

$s = 9.819$

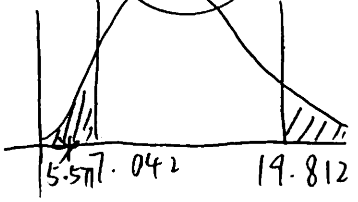
$\chi^2 = \dfrac{(14-1)(9.819)^2}{15^2}$

$= 5.571$

$p(\chi^2 < 5.571) = \chi^2\text{cdf}(0, 5.571, 13)$

$= 0.0396 < 0.10$

$\chi^2_R = 19.812$, $\cancel{\chi^2_L} = 7.042$

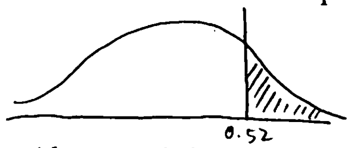


Reject H_0, there is sufficient evidence to support the claim at $\alpha = 0.10$ that the standard deviation of IQ scores of college students is less than 15.

(6) Use the given information to find the P-value.

12) a. Assume normal distribution. The test statistic in a right-tailed test is $z = 0.52$. Find p-value.

$p(z>0.52) = \text{normalcdf}(0.52, E99, 0, 1)$
$= 0.3015$ ✓



b. Assume t-distribution with n = 45. It is a two-tailed test. with test statistic t = –1.635. Find p-value

$p(t<-1.635)$ or $p(t>1.635) = 2 \times \text{tcdf}(-E99, -1.635, 44)$
$df = 44$ $\quad = 0.1092$ ✓

c. Assume chi-square distribution with n = 14. It is a left-tailed test and the test statistic is $X^2 = 6.278$. Find p-value.

$p(\chi^2 < 6.278) = \chi^2\text{cdf}(0, 6.278, 13)$
$n = 14, df = 13$ $\quad = 0.06455$ ✓

13) (7) In a random sample of 500 people aged 20–24, 110 were smokers. In a random sample of 450 people aged 25–29, 65 were smokers. Test the claim that the proportion of smokers age 20 –24 is higher than the proportion of smokers age 25 –29. Use a significance level of 0.01.

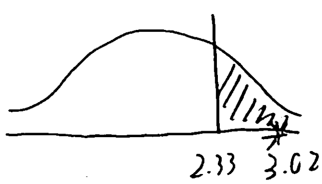
$n\hat{p} \geq 5$
$n\hat{q} \geq 5$
use normal dist

20-24	25-29
$n_1 = 500$	$n_2 = 450$
$x_1 = 110$	$x_2 = 65$
$\hat{p}_1 = 0.22$	$\hat{p}_2 = 0.144$
$\hat{q}_1 = 0.78$	$\hat{q}_2 = 0.856$

claim: $p_1 > p_2$
$H_0: p_1 = p_2$
$H_1: p_1 > p_2$ ✓

$z_{\frac{\alpha}{2}} = 2.33$

$\bar{p} = \dfrac{110+65}{500+450} = \dfrac{175}{950} = 0.184$ ✓



$\bar{q} = 1 - 0.184 = 0.816$

$z = \dfrac{0.22 - 0.144}{\sqrt{\dfrac{(0.184)(0.816)}{500} + \dfrac{(0.184)(0.816)}{450}}}$

$= 3.02$

Reject H_0, there is sufficient evidence to support the claim at $\alpha = 0.01$ that the proportion of smokers age 20-24 is higher than the proportion of smokers age 25-29. ✓

$p(z>3.02) = \text{normalcdf}(3.02, E99, 0, 1)$
$= 0.001264 \leq 0.01$

14) (7) In a random sample of 300 women, 49% favored stricter gun control legislation. In a random sample of 200 men, 28% favored stricter gun control legislation. Construct a 98% confidence interval for the difference between the population proportions $p_1 - p_2$. Can you conclude that proportion of woemen favored stricter gun control legislation is higher than that for men?

$n\hat{p} \geq 5$
$n\hat{q} \geq 5$
use normal

Women	men
$n_1 = 300$	$n_2 = 200$
$\hat{p}_1 = 0.49$	$\hat{p}_2 = 0.28$
$\hat{q}_1 = 0.51$	$\hat{q}_2 = 0.72$

$z_{\frac{\alpha}{2}} = 2.33$

$$E = 2.33\sqrt{\frac{(0.49)(0.51)}{300} + \frac{(0.28)(0.72)}{200}}$$

$= 0.09997$

$\hat{p}_1 - \hat{p}_2 = 0.49 - 0.28 = 0.21$

$0.11003 < p_1 - p_2 < 0.30997$

Since the interval is positive, $p_1 > p_2$. That is we can conclude that proportion of women favored stricter gun control legislation is higher than that for men.

15) (7) A coach uses a new technique in training middle distance runners. The times for 8 different athletes to run 800 meters before and after this training are shown below.

Athlete	A	B	C	D	E	F	G	H
Time before training (seconds)	118.7	111.1	115.1	109.4	117.9	111.3	116.2	109
Time after training (seconds)	119.3	109.8	112.7	110.2	116.1	111.4	112.6	105.1

Using a 0.05 level of significance, test the claim that the training helps to improve the athletes' times for the 800 meters. Assume samples have been randomly selected from normally distributed populations.

claim: $\mu_d > 0$
$H_0: \mu_d = 0$
$H_1: \mu_d > 0$

$n = 8$, $df = 7$

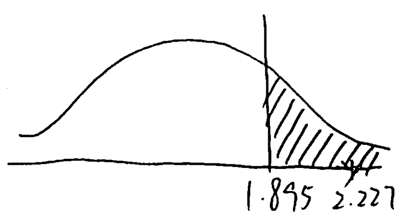
$\bar{d} = 1.4375$

$S_d = 1.826$

$$t = \frac{1.4375}{\frac{1.826}{\sqrt{8}}} = 2.227$$

$p(t > 2.227) = \text{tcdf}(2.227, E99, 7)$
$= 0.03062 \leq \alpha$

$t_{\frac{\alpha}{2}} = 1.895$



Reject H_0, there is sufficient evidence to support the claim at $\alpha = 0.05$ that the training helps to improve the athletes's times for the 800 meters.